CALCULATION OF THE TRANSVERSE FLEXURAL STRENGTH OF SHEET GLASS

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The equations of equilibrium between internal and external forces are used to derive relations and construct the computer program "Solid Glass" for performing practical calculations of the transverse flexural strength of sheet glass under a uniform load. The computed strength and deflection of sheet glass are in good agreement with the experimental data.

Key words: sheet glass, safety structures, strength calculation, computational program.

Primarily flat silicate glass with short-side/thickness ratio, as a rule, greater than 100 is used in light-transmitting designs and facade systems in buildings and structures. In real structures glass is secured to the structural elements on four sides by metal support strips through rubber sealing gaskets. For purposes of theoretical investigations such securing can be regarded as partial clamping of a plate on four sides.

Since under transverse bending tensile stresses are present on the median surface at the center of the sheet, and the deflection in the limiting state is greater than the thickness of the sheet, we shall consider such a structure to be a flexible plate.

At the present time, the methods of the theory of elasticity are used to determine the strength of such plates, including the method of rows, the method of grids, and the energy method. B. G. Galerkin, I. G. Bubnov, P. M. Varvak, S. P. Timoshenko, A. S. Vol'mir, and Yu. A. Shimanskii and others have made a large contribution to the theory of plates. The strength of plates was mainly determined by solving comparatively complicated differential equations with boundary conditions applicable to thick or thin plates, for which the ratio of the edge length to the thickness was less than 100. Under such conditions the maximum stresses in the limiting state arose at the center of the plate.

The following boundary conditions were used in the present work:

the theory of the highest normal stresses is used for silicate glass; accordingly, fracture under bending occurs because the tensile stresses reach limiting values;

the hypothesis of flat cross sections remains valid in glass under bending; accordingly, the stress over the thickness of a sheet varies according to a linear law;

the ratio b/h of the length of the short side of a plate to its thickness ranges from 100 to 300; and,

the surfaces of the plate are parallel to one another.

Experiments, whose results are presented in work by N. V. Kondrat'eva [1], have established that for transverse bending the deformation appearing at the center of the plate as a result of tension and compression was much smaller than the deformations determined in other zones. Therefore, in flexible plates under transverse bending by a uniform load the stresses at the center of a plate prior to fracture do not reach their limits for the indicated values of b/h.

Analysis of the changes of the deformation along the x and y axes and the diagonals of a plate [1] shows that the maximum deformation at fracture of the samples is observed in the corner zones at a distance equal to approximately 1/8 times the length of the diagonal (Fig. 1).

Since glass in the compressed and stretched zones is in a complicated stressed state, and the thickness of the plate is much smaller than its other dimensions, we shall assume that there is no reciprocal pressure from the layers. We shall assume that the normal stress σ_z in the direction of the thickness of the plate and the tangential stresses are small compared with the normal stresses σ_x and the stresses σ_y parallel to the surface of the plate.

We shall use the method of equilibrium between the internal and external forces in a section to assess the stressed state of glass in a plate. A. A. Gvozdev developed and successfully used this method to calculate reinforced concrete

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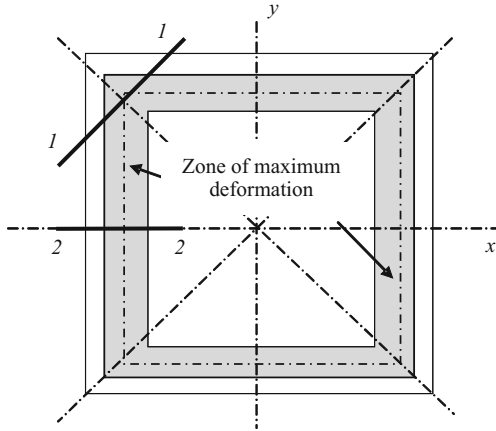
Fig. 1. Zone of maximum deformations in a plate under transverse bending under a uniform load.
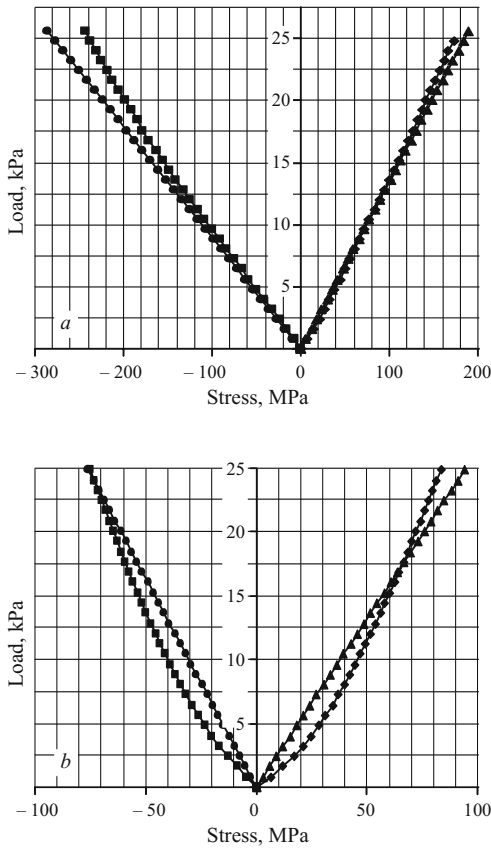

Fig. 3. Deflection at the center of the glass samples: *1* and *2*) rows 1 and 2; row 1) experimental data; row 2) theoretical values.

Fig. 2. Experimental and theoretical values of the stresses in 1500 × 1500 × 6 mm samples (4 – 12% error with fracture): *a*) section *1 – 1*; *b*) section *2 – 2*; ◆) row 1; ■) row 2; ▲) row 3; ●) row 4; rows 1 and 2) experimental data; rows 3 and 4) theoretical values.

structures in a limiting state. The basic equation of the method expresses the equality between the external forces acting on an element of the plate and the internal forces arising in this element.

Analysis of the curves of the maximum deformations of the glass in the corner zone along a diagonal and in the zone
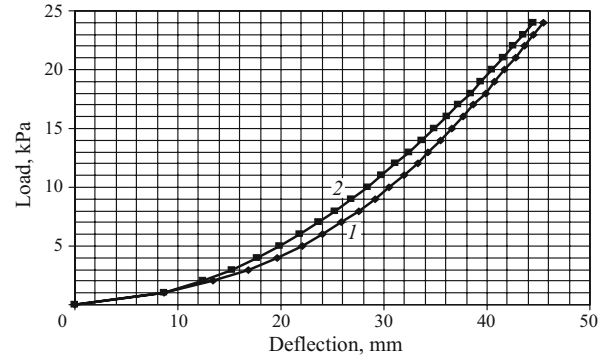
near the support on the *x* and *y* axes shows that the glass in the corner zone (section *1 – 1*) perpendicular to the diagonal (see Fig. 1) absorbs the load from bending and compression. The neutral axis is located at distance 0.4 times the thickness of the glass from the surface of the stretched zone.

Deformations from bending and stretching arise in a zone along the *x* axis in the section *2 – 2*. The neutral axis is located at distance 0.55 times the thickness of the glass from the stretched zone.

The results obtained were used as a basis to develop a computational scheme for a flexible glass plate to calculate transverse bending under a uniform load. The equations for the internal forces in elements of unit width in the sections *1 – 1* and *2 – 2* were constructed taking account of the stress distribution obtained experimentally.

Equating the external forces acting on an element and the internal forces gave the following:

for the section *1 – 1*:

$$\sigma_t = \left(\frac{a}{b}\right)^{\beta} 0.68 kcq\alpha^2 \ \frac{b^2}{h^2} \ ; \ \sigma_c = \left(\frac{a}{b}\right)^{\beta} 1.02 kcq\alpha^2 \ \frac{b^2}{h^2} \ ;$$

for section *2 – 2*:

$$\sigma_t = \left(\frac{a}{b}\right)^{\beta} 0{,}67 kcq\alpha^2 \ \frac{b^2}{h^2} \ ; \ \sigma_c = \left(\frac{a}{b}\right)^{\beta} 0.54 kcq\alpha^2 \ \frac{b^2}{h^2} \ ;$$

deflection at the center:

$$f = a \times 10^{-4} P^{\gamma} 12(1 - \mu^2),$$

where σ_t and σ_c are the total tensile and compression stresses in the glass, MPa; *a* and *b* are the lengths of the long and short sides of the plate; mm; *h* is the thickness of the plate, mm; β is a coefficient that accounts for the effect of the edge ratio on the stressed state of the plate; *k* is a coefficient that takes account of the nonuniform distribution of the bending moment along the perimeter of the plate; *c* is a coefficient that accounts for the compliance of the supports; *q* is the uniformly distributed load, kPa; α is a coefficient that accounts

for the type of support for the plate (cantilever, clamped, or partially clamped); f is the deflection at the center of the plate, mm; $P = \dfrac{q}{E} \times 10^{-1} \, (b/h)^3$ (E is Young's modulus, MPa); γ is a coefficient that accounts for the change of the plate's stiffness; and, μ is the coefficient of transverse deformation.

These relations were used to calculate the tested samples of sheet glass with different edge ratio and short edge-to-thickness ratio. The theoretical values of the stresses are in good agreement with the experimental values. The error did not exceed 15%.

The computer program "Solid glass" was developed, on the basis of the results obtained in the present investigations, to calculate the strength of sheet glass under transverse bending. This program is used to design light-transmitting coating structures, coatings, and facade systems. To ensure that the designs are safe, the program permits introducing the appropriate reliability coefficients taking account of the number of floors and capacity of the building (up to 100 floors). This program has been used to calculate light-transmitting facade systems for several tall buildings, glass covers, and portholes on ships. The theoretical and experimental values of the stresses in the glass and the deflections are presented in Figs. 2 and 3.

The computational method proposed here and the "Solid glass" computer program can be used to calculate sheet glass when designing light-transmitting safety and bearing structures.

REFERENCES

1. N. V. Kondrat'eva, "Experimental investigations of the strength of sheet glass under transverse bending," *Steklo Keram.*, No. 2, 5 – 7 (2006).